

FREEWAY TRAFFIC SPEED ESTIMATION USING SINGLE LOOP OUTPUTS

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Abstract - Traffic speed is one of the most important indicators for traffic control and management. Unfortunately, speed can not be measured directly from single inductance loops, the most commonly used detectors. To calculate space-mean speed, a constant g is often adopted to convert lane occupancy to traffic density. However, as will be illustrated by our data in this study, such a formula consistently underestimates speed whenever a significant number of trucks and/or other longer vehicles are present. This is due to the fact that the g value is actually not a constant, but rather a function of vehicle length. To calculate the g value suitably, we need to know the long vehicle (LV) percentage or mean vehicle length in real-time. However, such information is not directly available from single loop outputs. In this paper, we show how the occupancy variance obtained from single loop data can be used to estimate long vehicle percentage, and how a log linear regression model for mean vehicle length estimation based only on single loop outputs can be developed. The estimated mean vehicle length is used to calculate the corresponding g value in real-time in order to estimate speed more accurately. Our speed estimations with corrected g are very close to the speeds observed by the speed trap in the current study.

Key words: speed estimation, single loop data, long vehicle percentage, effective vehicle length, lane occupancy variance

INTRODUCTION

Traffic speed is one of the most desirable variables for real-time traffic control and traveler information systems because it is both a potential indicator of problems on the roadway and also a good measure of system effectiveness. However, traffic speed data are not directly measured by detectors for most highway systems. This makes it an important goal for transportation researchers and managers to obtain traffic speed from available measurements.

As mentioned by Dailey (1), many highway management systems use inductance loops to gather

volume (the number of vehicles passing per unit time) and occupancy (the fraction of some total time that a vehicle occupies a loop) data of each lane. Most of these systems rely on single loops spaced at large distances rather than loops placed in pairs at a short distance. Such single loop detectors are able to measure volume and occupancy, but not speed. Speed must be estimated from the measured variables, i.e. volume and occupancy of each lane. The estimation accuracy depends largely on the quality of the measurements as well as the traffic condition (i.e., traffic composition).

A commonly adopted method is to use a constant g to convert occupancy to density, and use the fundamental speed, volume, density formulation to calculate space-mean speeds as shown in Equation (1) where density is given as a function of lane occupancy. The estimation inputs are single loop outputs, $N(i)$ (volume) and $O(i)$ (occupancy) of the i th time interval.

$$\bar{s}_s(i) = \frac{N(i)}{T \cdot O(i) \cdot g} \quad (1)$$

Where i = time interval index,

\bar{s}_s = space mean speed in mph for each interval;

N = vehicles per interval (volume),

O = percentage of time loop is occupied per interval (lane occupancy), and

T = hours per interval (e.g., for the current study $T = 1/12$ hour or 5 minutes).

In Equation (1), $\bar{s}_s(i)$ represents space-mean speed of the i th time interval, and g is the constant, which converts the units to their proper values and is related to mean vehicle length plus detector size. Equation (1) was first developed in the 1960s [see (2)], and more clearly presented by subsequent researchers such as Mikhalkin *et al* (3) Gerlough *et al* (4) and Courage *et al* (5). Due to its simplicity, many freeway systems use Equation (1) to estimate speed from single loop outputs (6 - 7).

Regarding the fitness of Equation (1), there are different opinions. On the one hand, Hall *et al* (6) and Pushkar *et al* (8) collected data from several stations,

plotted g versus occupancy, and found that g value is not constant but varies with occupancy. They also investigated the procedure for deriving Equation (1) and concluded that the assumptions that make g a constant are normally hard to meet. On the other hand, Coifman (9) did similar work but got an opposite conclusion, that g is very close to a constant. Whether or not we can use Equation (1) to estimate speed remains an interesting topic.

In addition to studies that use Equation (1) for speed estimation, some other methodologies have also been developed. Pushkar *et al* (8) developed a cusp catastrophe theory model to estimate speed. The comparison of the estimation results between their model and Equation (1) indicated that the cusp catastrophe theory model gave more reasonable results. Dailey (10) considered random errors in the measurements and used a Kalman filter to estimate speed. The estimation results were basically consistent with the observed speeds, but with a smaller variance. To apply the aforementioned models, several parameters must be calibrated, and the calibrations require information beyond the measurements of single loops.

In this paper, we will focus on the fitness of using Equation (1) to estimate freeway traffic speed. Data used in this study is briefly addressed at first, followed by a discussion of g and its determinants. Then, based on the theoretical derivation of the occupancy and effective vehicle length relationship, a log linear model for mean effective vehicle length estimation is presented. The estimated mean effective vehicle lengths are used to calculate the g value of each time interval in order to get a more accurate speed estimation. The effectiveness of using this corrected g for speed estimation will be summarized in the last part of this paper.

DATA

The Washington State Department of Transportation (WSDOT) has a network of traffic counters embedded in the roadway. These traffic sensors are 182.88cm (6 feet) wide square loops of copper wire connected to cabinets located beside the road. They are located about every half-mile on mainline lanes and ramps of freeways and state highways in the central Puget Sound region, including I-5, I-405, I-90, SR520, SR18, SR522 and SR99 (7). When a vehicle drives over a loop, it is counted, and the time that the vehicle spends over the loop is measured. The data are transmitted every 20 seconds to the WSDOT Traffic Systems Management Center (TSMC). In other words, the WSDOT's freeway

loop detectors do not produce data for each specific vehicle, but rather for all vehicles aggregately within 20-second intervals for processing and archiving. Based on these outputs, we can further aggregate the data to any desired longer time interval. In our speed estimation for this paper, we use data aggregated to 5-minute intervals (i.e. 15 consecutive 20-second values are combined to produce a single 5-minute value).

At Station ES-167D (under 145th Street's over bridge of I-5), there are three general purpose lanes and one HOV lane on southbound I-5. Speed traps, two consecutive loops several feet apart, are imbedded in each lane for measuring vehicle lengths and traffic speeds in addition to volume and occupancy. Loop data collected for 24 hours on Thursday, May 13, 1999, were applied for analysis and model estimation. There was no incident detected that day. Loop_MS__2 (the WSDOT uses exactly 7 characters as loop code to indicate the loop's location and purpose. See (7) for details) was used to collect volume and occupancy for the middle general purpose lane. Vehicle lengths, measured by the speed trap (The pair loops _MS__2 and _MS__S2), were used for the estimation of our proposed effective vehicle length model. And the speed trap measured speeds were applied for the verification of the predictions using our calculated g .

Descriptive statistics of the volume data measured by _MS__2 and _MS__S2 are given in Table 1. We can see that the volume count difference between the two loops is very small, indicating a good quality of the raw data. All these data are available in electronic form at the University of Washington's ITS website (http://www.its.washington.edu/tdad/tdad_top.html).

TABLE 1 Descriptive Statistics of Loop Measured Volume at 20-Second Interval

Loop code	_MS__2	_MS__S2	Difference (percentage)
Minimum	0	0	0 (0.0%)
Maximum	18	17	1 (5.5%)
Mean	6.73	6.71	0.02 (0.3%)
Standard Deviation	3.86	3.83	0.03 (0.8%)
Total count for 24 hours	29061	29006	55 (0.2%)

DETERMINANTS OF g

As aforementioned, researchers disagree about whether g is a constant in Equation (1). In this section, we will show what g is, what determines its value and under what circumstances it can vary significantly.

If during the i th interval of our analysis, $N(i)$ and $O(i)$ are observed, then the following relationship exists,

$$O(i) = \frac{100}{5280T} \sum_{j=1}^{N(i)} \frac{l_j(i)}{s_j(i)} \quad (2)$$

where $l_j(i)$ and $s_j(i)$ is the j th vehicle's effective vehicle length (the sum of actual vehicle length and detectable length of loop detector in feet) and speed in miles per hour, respectively, in interval i . T is equal to total hours per interval. If we can assume that all vehicles in interval i have uniform length, then we can treat the mean effective vehicle length, $\bar{l}(i)$, as a constant and get

$$O(i) = \frac{\bar{l}(i)}{52.8T} \sum_{j=1}^{N(i)} \frac{1}{s_j(i)} \quad (3)$$

If we use $\bar{s}_s(i)$ to represent the space-mean speed, i.e.,

$$\bar{s}_s(i) = \frac{N(i)}{\sum_{j=1}^{N(i)} \frac{1}{s_j(i)}} \quad (4)$$

equation (3) can be rewritten as

$$\bar{s}_s(i) = \frac{N(i) \cdot \bar{l}(i)}{52.8T \cdot O(i)} \quad (5)$$

Then Equation (5) and Equation (1) yield

$$g = \frac{52.8}{\bar{l}(i)} \quad (6)$$

Equation (6) shows that the g value depends on the mean effective length of vehicles. If $\bar{l}(i)$ does not change with time, then g will be a constant. However, under at least some of the traffic situations, $\bar{l}(i)$ varies significantly over time. Therefore, Equation (6) should be written in a more general form,

$$g(i) = \frac{52.8}{\bar{l}(i)} \quad (7)$$

Table 2 shows the descriptive statistics of observed speeds and vehicle lengths in our data set with suspected data excluded. Of the 4120 20-second observations, 116 had no vehicles present and 1069 were flagged as suspected errors. The standard deviation of vehicle length of all the qualified cases was 9.33 feet (2.85 meters) as shown in Table 2. This is about 46.5% of the mean. If we further exclude the cases with trucks and other long vehicles present, the standard deviation reduced to 1.31 feet or only 8.63% of the mean; the latter indicates a traffic situation which reasonably meets the uniform length assumption.

Figure 1 shows the comparison of observed speeds and estimated speeds using a constant $g = 1.928$ at 5 minute intervals. Normally the WSDOT uses $g = 2.4$ for speed estimation (7). However, for our data set, if we use $g = 2.4$, the estimated space-mean speed is about 9% lower than the observed space-mean speed (converted from the observed 20 second speed data). To optimize our estimation, we use the constant g value that results in the same mean between the estimated and the observed speeds. The correlation coefficient between the estimated mean speeds and the observed mean speeds is 0.64. This is the best estimation we can get through Equation (1) using a constant g , but the bias is still very clear, especially the underestimation between 1:00am to 4:00am. If we also check the LV percentage curve that is plotted in Figure 1, we can find that the fluctuation of the curve is basically consistent with the differences between the speed curves. The correlation between the speed differences and LV percentage is as high as 0.92. Hence, the speed estimation with fixed g value has bias. When the LV percentage is higher than the average, Equation (1) underestimates speeds, and vice versa.

TABLE 2 Descriptive Statistics of Speed Trap Observed Mean Speeds and Mean Vehicle Lengths per 20-Second Interval

	Speed (mph ^a) (2887 observations)	Length (feet ^a) (2887 observations)	Length (feet ^a) (1657 observations without LVs ^b present)
Minimum measured	16.70	11.00	11.00
Maximum measured	82.00	88.00	25.00
Mean of all the observations	62.43	20.05	15.18
Standard deviation	6.16	9.33	1.31
SD/Mean (%)	9.87	46.53	8.63

^a 1 mile = 1.609 kilometers and 1 foot = 0.305 meters

^b The definition of long vehicle (LV) here is the vehicles with measured length over 7.93m (26 feet).

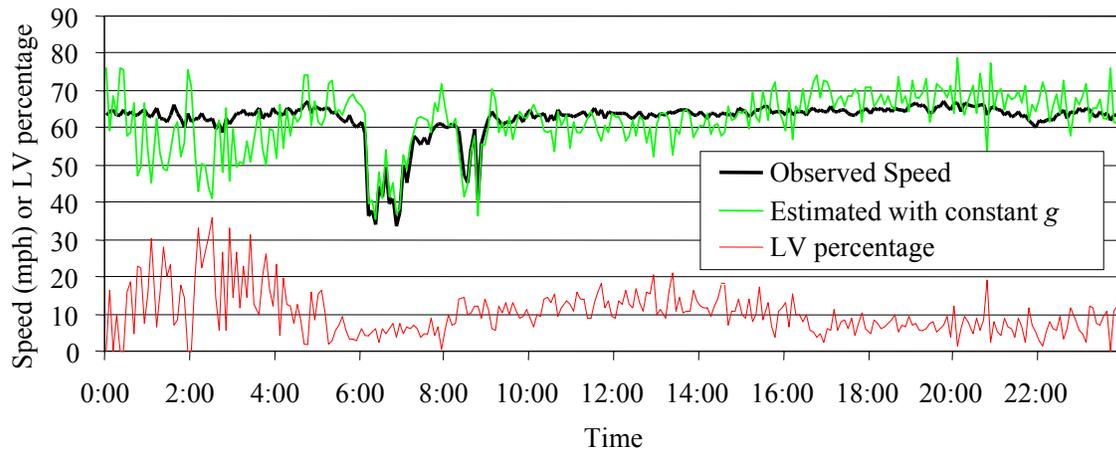


FIGURE 1 Comparison of Observed Speeds and Estimated Speeds by Equation (1) with a Constant g

As an example, let's consider an extreme case. In one time interval, only one 3-meter-long (equivalent to 9.84 feet long) car is observed, and in the next time interval, a 21-meter-long (equivalent to 69 feet long) truck passed over the loop. Even if the two observed vehicles were traveling at the same speed, the estimation results using a constant g can give a car speed that is 6 times higher than the truck speed.

Such estimation bias may be corrected by using the proper g value for each time interval. Instead of using a constant g , we calculate the g value at each time interval as a function of the observed effective vehicle length, and the estimated result is shown in Figure 2. The estimated speed curve is very close to the observed speed curve. The corresponding correlation coefficient

is increased to 0.94. This result is fairly ideal considering the dynamic features of vehicle movements. Therefore, we can get a favorable speed estimation through Equation (1) if the appropriate g value for each time interval is used.

To calculate the g value suitably, we need to know the long vehicle (LV) percentage or mean vehicle length in real-time. Although such information can be obtained from speed traps, it is not directly available from single loop outputs. To obtain good speed estimates for all freeway stations, efforts are needed to get the desired information from single loop stations as well. In the following section, we will show how the mean vehicle lengths can be estimated from single loop outputs.

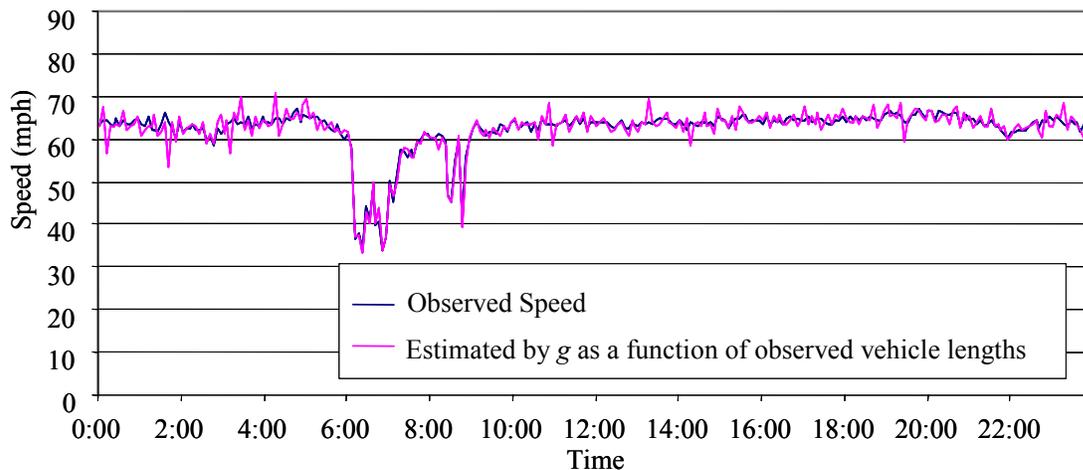


FIGURE 2 Estimation Results Using g Values Calculated as a Function of Observed Vehicle Lengths

ESTIMATION OF EFFECTIVE VEHICLE LENGTH FOR SINGLE LOOPS

The Relationship of Effective Vehicle Length and Occupancy

As the length of a LV is significantly longer than that of a normal passenger car (PC), defined as vehicles with length shorter than 7.93m (26 feet), its mix rate with PCs will definitely affect the accuracy of speed estimation by Equation (1). To evaluate such effects, let's consider the statistical features of the variables in our formulations.

$$O(i) = \frac{1}{52.8T} \sum_{j=1}^{N(i)} \frac{l_j(i)}{s_j(i)} + \varepsilon(i) \quad (8)$$

where $\varepsilon(i)$ is the error term with mean 0. Then the expectation of $O(i)$ is

$$E(O(i)) = \frac{1}{52.8T} E\left(\sum_{j=1}^{N(i)} \frac{l_j(i)}{s_j(i)}\right) = \frac{N(i)}{52.8T} E\left(\frac{l_j(i)}{s_j(i)}\right) \quad (9)$$

As our data indicate, the correlation coefficient between $l_j(i)$ and $1/s_j(i)$ is only -0.096 , therefore we treat the two as independent random variables. Then we have

$$E(O(i)) = \frac{N(i)}{52.8T} E(l_j(i)) E\left(\frac{1}{s_j(i)}\right) \quad (10)$$

Considering the random error $\Delta s_j(i)$ in speed measurement, and using power series expansion, we get

$$E\left(\frac{1}{s_j(i)}\right) = \frac{1}{s(i)} \cdot E\left(\frac{1}{1 + \frac{\Delta s_j(i)}{s(i)}}\right) = \frac{1}{s(i)} \cdot E\left(1 - \frac{\Delta s_j(i)}{s(i)} + \frac{\Delta s_j^2(i)}{s^2(i)} - \dots\right) \quad (11)$$

Where $\bar{s}(i)$ is the time mean speed for interval i . Omit the items with higher power, and notice that $E(\Delta s_j(i))=0$ and $E(\Delta s_j^2(i)) = \sigma_s^2(i)$, we can obtain,

$$E\left(\frac{1}{s_j(i)}\right) = \frac{1}{s(i)} \left(1 + \frac{\sigma_s^2(i)}{s^2(i)}\right) \quad (12)$$

Substitute Equation (12) into Equation (10), we can get

$$E(O(i)) = \frac{N(i) \cdot \bar{l}(i)}{52.8T \cdot \bar{s}(i)} \left(1 + \frac{\sigma_s^2(i)}{s^2(i)}\right) \quad (13)$$

Dailey (10) used Equation (13) to calculate the mean speed, assuming that the occupancy measurements were perfect, and the mean vehicle length and speed variance were known. Dailey also concluded that such estimation has less bias than estimation by Equation (1). In reality, however, both the mean vehicle length and speed variance of each time interval are unknown and they may vary a lot from time to time.

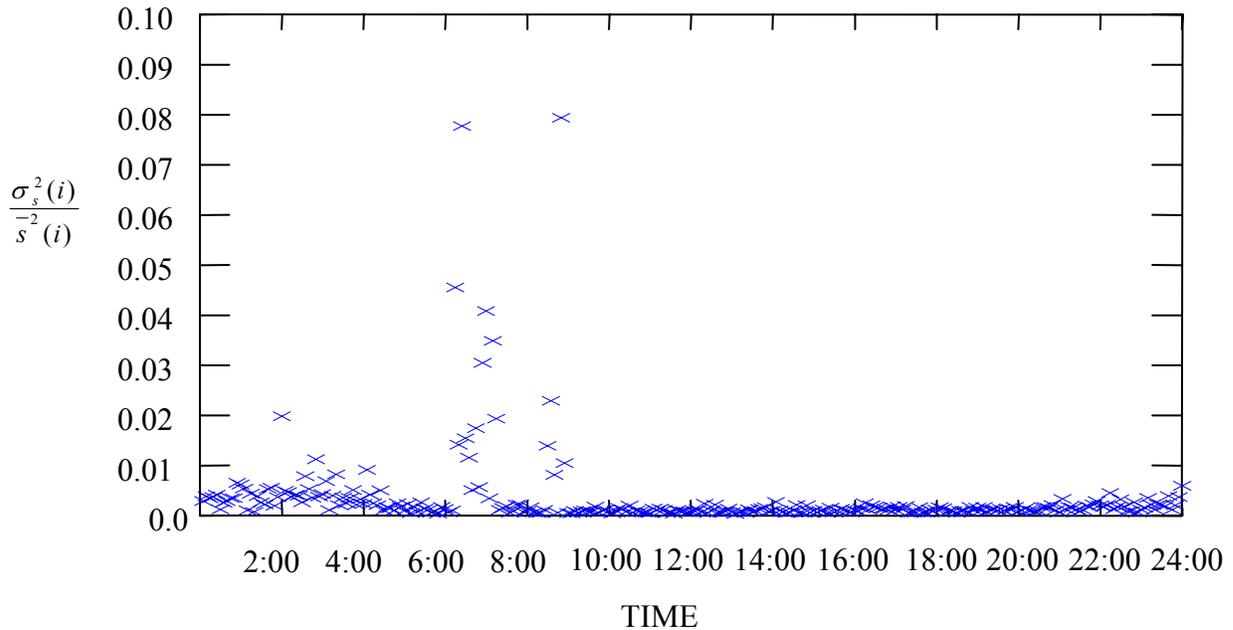


FIGURE 3 Speed Variance and Mean Speed Square Comparison

Figure 3 shows the change of $\sigma_s^2(i)/\bar{s}^2(i)$ with time. For most of the intervals, $\sigma_s^2(i)/\bar{s}^2(i)$ is smaller than 0.01. Even for the exceptional case with the largest value of $\sigma_s^2(i)/\bar{s}^2(i) = 0.079$, the contribution of $\sigma_s^2(i)/\bar{s}^2(i)$ to the occupancy mean is still much smaller than that of mean vehicle length. Therefore, to simplify our analysis, we neglect the effect of speed variance and assume all the vehicles travel at the same speed within each time interval (for short time intervals, i.e. 5 minutes or less). This makes the time mean speed ($\bar{s}(i)$) equal to the space mean speed ($\bar{s}_s(i)$) for each time interval. Then a simplified form of Equation (13) can be expressed in Equation (14).

$$E(O(i)) = \frac{N(i) \cdot \bar{l}(i)}{52.8T \cdot \bar{s}(i)} \quad (14)$$

Following a similar procedure, and omitting the tiny effect of speed variance, we can get the occupancy variance $V(O(i))$ as

$$V(O(i)) = \frac{N^2(i) \cdot \sigma_l^2(i)}{52.8^2 \cdot T^2 \cdot \bar{s}^2(i)} \quad (15)$$

where $\sigma_l^2(i)$ is the effective vehicle length variance of interval i . Combining Equations (14) and (15) to get rid of the speed term, results in Equation (16)

$$\bar{l}^2(i) = \frac{E^2(O(i)) \cdot \sigma_l^2(i)}{V(O(i))} \quad (16)$$

Equation (16) shows the relationship between effective vehicle length and occupancy moments with speed effect excluded. If we assume that there are $n_c(i)$ PCs and $n_l(i)$ LVs present in interval i , then the mean effective vehicle length can be expressed as

$$\bar{l}(i) = \frac{n_c(i) \cdot \bar{l}_c(i) + n_l(i) \cdot \bar{l}_l(i)}{N(i)} = \bar{l}_c(i) + x(i) \cdot (\bar{l}_l(i) - \bar{l}_c(i)) \quad (17)$$

where $x(i) = n_l(i)/N(i)$ is the LV percentage, $\bar{l}_c(i)$ and $\bar{l}_l(i)$ are mean length of PCs and LVs respectively for time interval i . Similarly, we can get the variance of vehicle length expressed by LV percentage $x(i)$,

$$\sigma_l^2(i) = x(i) \cdot (1-x(i)) \cdot (\bar{l}_l(i) - \bar{l}_c(i))^2 + \sigma_c^2(i) + x(i) \cdot (\sigma_l^2(i) - \sigma_c^2(i)) \quad (18)$$

where $\sigma_l^2(i)$ and $\sigma_c^2(i)$ are the effective length variances of PCs and LVs respectively. Equations (17) and (18) reflect how the LV percentage affects mean effective vehicle length and variance, and explain why, in Figure 1, LV percentages were correlated with the

differences between the observed speeds and the estimated speeds by using a constant g .

Mean Effective Vehicle Length Estimation

As has been proven earlier in this paper, real-time effective vehicle length information is the key for more accurate speed estimation using Equation (1). If we use single loop outputs to estimate the mean effective vehicle length of each time interval, our speed estimation accuracy using single loop data can be significantly improved.

Based on Equation (16), we know that the mean effective vehicle length is a function of $E(O(i))$, $V(O(i))$ and $\sigma_l^2(i)$. Both $E(O(i))$ and $V(O(i))$ can be calculated from measured occupancies, but $\sigma_l^2(i)$ is not directly available. Based on Equation (18), $\sigma_l^2(i)$ should be related to the vehicle composition, i.e., LV percentage, and LV percentage is related to the time of day and traffic volume. Therefore, besides the occupancy moments, we use three more variables, traffic volume, low flow dummy (LFD , 1 when hourly volume is lower than 300, and 0 otherwise), and high flow dummy (HFD , 1 when hourly volume is higher than 1680, and 0 otherwise), to explain $\sigma_l^2(i)$. A log linear regression model was formed as

$$\ln \bar{l}(i) = \beta_0 + \beta_1 \cdot [2 \ln E(O(i)) - \ln V(O(i))] + \beta_2 \cdot \ln N(i) + \beta_3 \cdot HFD + \beta_4 \cdot LFD + \varepsilon \quad (19)$$

where β_0 , β_1 , β_2 , β_3 and β_4 are estimation coefficients, and ε is the error term with $E(\varepsilon) = 0$.

Speed-trap measured vehicle lengths were used to calibrate the model shown in Equation (19). Ordinary Least Squares method was used in the model estimation. The estimated coefficients and their significance levels (indicated by t -ratios) are given in Table 3. All the selected variables in the model are significant at the 0.01 level. Figure 4 gives a comparison of the observed and estimated mean vehicle length.

Please note that it is important that the terms $2 \ln E(O(i))$ and $-\ln V(O(i))$ have the same coefficient β_1 in order to eliminate, or at least minimize, the speed effects on mean effective vehicle length estimation.

TABLE 3 Estimation Results of Equation (19)

Sample number: 288 $R^2 = 0.47$			
Variable name	Coefficient Symbol	Estimated Coefficient	t-ratio
CONSTANT	β_0	3.238	53.41
Occupancy moments ($2LnE(O(i)) - LnV(O(i))$)	β_1	-0.068	-11.20
Volume ($Ln(N(i))$)	β_2	0.059	4.26
High flow dummy (<i>HFD</i>)	β_3	-0.024	-2.88
Low flow dummy (<i>LFD</i>)	β_4	0.136	5.20

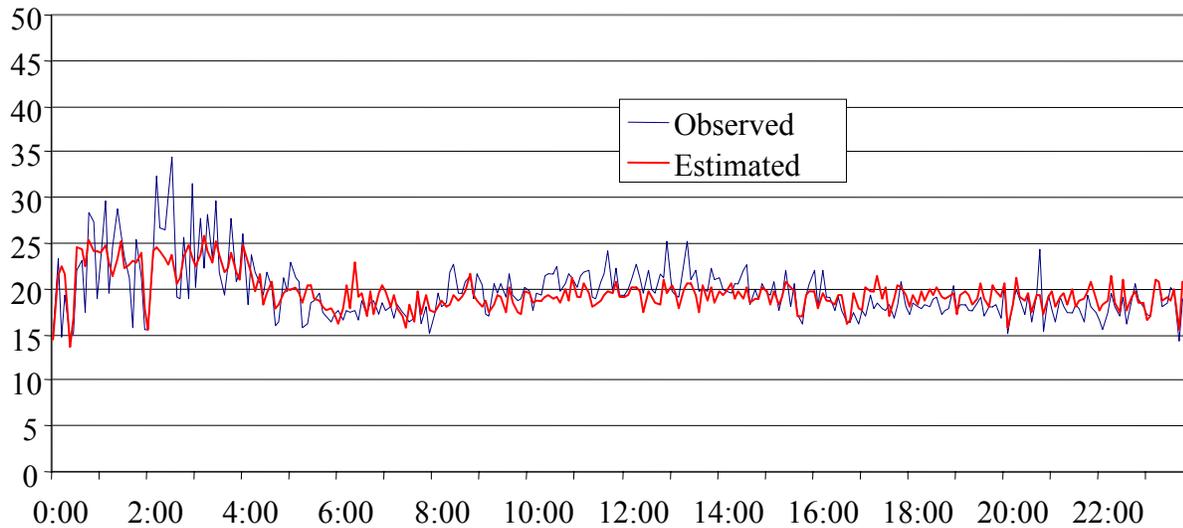


FIGURE 4 Comparison of Observed Mean Vehicle Lengths and Estimated Mean Vehicle Lengths

SPEED ESTIMATION WITH CORRECTED g

With the estimated effective mean vehicle length, we can calculate the corrected g value for each time interval, according to Equation (7). This g value for each time interval is shown in Figure 5. The g value is obviously lower during the 1:00am to 4:00am time period, which corresponds to the period with higher LV percentage indicated by Figure 1.

Again, we use Equation (1) to estimate the speed for each interval. Instead of using a constant g value, we use the corrected g value for each interval calculated by

Equation (7). The estimation result is plotted in Figure 6. Comparing this with the constant g based speed estimation shown in Figure 1, the underestimation or overestimation bias has been significantly reduced by using the corrected g . Our current estimation result is basically consistent with the observed speeds, but varies more.

The comparison of speed estimation using a fixed g value and speed estimation using a g value that is a function of mean effective vehicle length is summarized in Table 4. The improvement of estimation accuracy shows that our proposed method is an improvement over existing procedures.

TABLE 4 Comparison of Speed Estimation Results

Estimated by using	Fixed g (WSDOT)	Estimated g values (Proposed Method)	Estimated g values (Based on Observed Vehicle Lengths)
R^2	0.41	0.59	0.89
Standard error of estimation	4.17	3.47	1.76

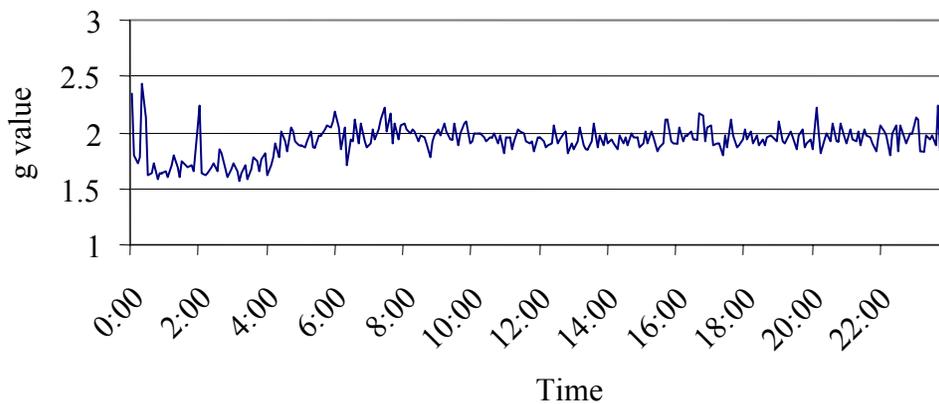
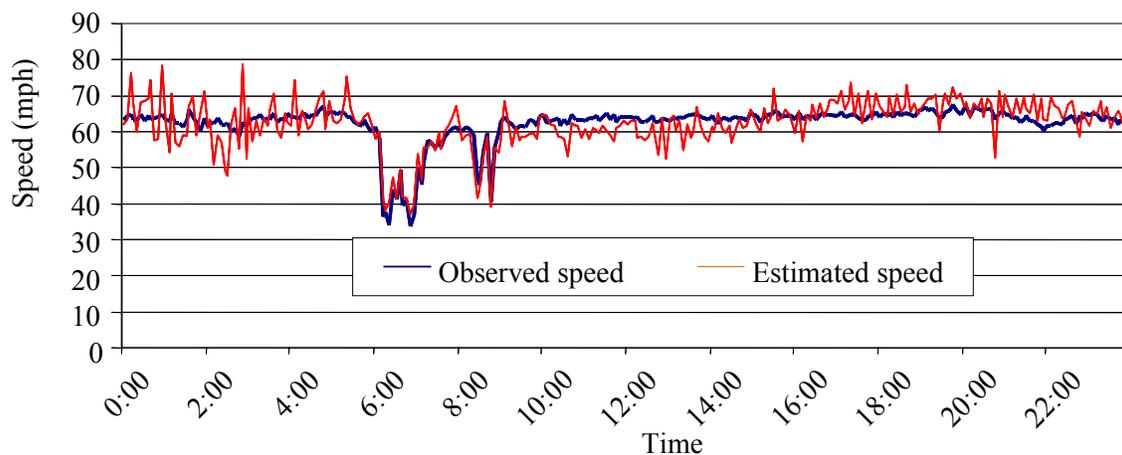
FIGURE 5 The Estimated g Value for Each Time Interval

FIGURE 6 Comparison of Observed Speeds and Estimated Speeds by the Proposed Method

CONCLUSION

To meet the needs of dynamic traffic control and management, and advanced traveler information systems, providing real-time speed information to road users is becoming more and more important. Being one of the most common detectors on highway systems, single inductance loops produce abundant real-time data. To estimate traffic speeds from single loop outputs is very desirable.

The traditional speed estimation method using g as a constant over time produces biased estimations, especially when truck percentages are high. However, this does not mean Equation (1) is unsuitable for speed estimation. As has been demonstrated in this paper, by choosing proper g values, Equation (1) can give very good estimations.

The speed estimation parameter, g , changes with the mean of effective vehicle length for each time interval. To minimize the bias in speed estimation by Equation (1), we need to calculate the g value at each time interval based on the real-time mean of the effective vehicle length. Although vehicle length is not directly available, our statistical analysis has shown that it is estimable from the occupancy moments and other measured variables. Based on our statistical inference, a log linear regression model for effective vehicle length estimation is developed.

A commonly encountered difficulty for calibrating such regression models is how to get rid of the effect of speeds on effective vehicle length estimation. In this study, we minimized the possible speed effect by aggregating the loop outputs to 5-minute intervals and properly utilizing the statistical moments of the measured occupancies. The estimated mean of effective

vehicle length was used to calculate the g value for speed estimation using Equation (1), and this led to a more accurate speed estimation. Although there is still some bias in our results, this proposed method largely reduced the estimation bias caused by using a constant g . The method developed in this study does not require information from more than single loop outputs; this makes it easier for general site application.

For the application of this methodology, several issues need to be clarified which are topics for future research. Our first concern is the transferability of the developed model. Data from different stations with different geometric features are needed to test the variation of the model's coefficients. If the model's coefficients are significantly affected by geometric features, we may have to include geometric factors in our model, or use different models for freeways with different geometric features. Another concern is the optimal time interval for this methodology. To increase the interval, we can get more accurate estimations of the occupancy mean and variance, but the uniform speed assumption may easily be violated. Hence further studies are required to address these interesting issues specifically.

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