A Feedback-Based Dynamic Tolling Algorithm for High Occupancy Toll (HOT) Lane Operations

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ABSTRACT

Dramatically increasing travel demands and insufficient traffic facility supplies have induced severe traffic congestion. High Occupancy Toll (HOT) lane operation has been proposed as one of the most applicable and acceptable countermeasures against freeway congestion. By balancing pricing and vehicle occupancy constraints, HOT lane operations can realize the optimal traffic allocation and enhance overall infrastructure efficiency. However, few previous studies have concentrated on optimal tolling strategies. Two major problems with the inferior tolling strategies degrade HOT lane system performance. First, the under-sensitive tolling algorithm is incapable of handling the hysteresis properties of traffic systems and may cause severe response delays. Secondly, unfavorable flow fluctuation on HOT and GP lanes may be generated agitating traffic operations due to the over-sensitive characteristics of the imperfect tolling strategies. To address these problems, we develop a new feedback-based tolling algorithm to optimize HOT lane operations. To decompose the calculation complexity, a second-order control scheme is exploited in this algorithm. Based on traffic speed conditions and toll changing patterns, the optimum flow ratio for HOT lane utilization is calculated using feedback control theory. Then the appropriate toll rate is estimated backward using the discrete route choice model. This algorithm is simple, effective, and easy to implement. VISSIM-based simulation tests were conducted to examine its practicality and effectiveness. The test results show that the proposed tolling algorithm performed reasonably well in optimizing overall traffic operations of the HOT lane system under various traffic conditions.

Key words: High Occupancy Toll (HOT) lane, dynamic tolling algorithm, feedback control theory, and HOT lane simulation.

1. INTRODUCTION

In the last two decades, dramatically increasing travel demands and insufficient traffic facility supplies have induced severe traffic congestion. Traffic congestion costs billion of dollars every year, including lost time, wasted energy, excess air pollution, and lost productivity. The 2005 Urban Mobility Report indicates the annual delay per person was 47 hours and an average of $794 per traveler resulted from congestion in the 85 surveyed-urban areas in 2003 *(1)*. It is of practical importance to manage and utilize the existing traffic facilities more efficiently when expanding highway capacities becomes more difficult in metropolitan areas. High Occupancy Vehicle (HOV) lane has been a solution for several decades. The concept was originally proposed as dedicated bus lanes. Gradually, HOV lane allowed vanpools and carpools. It has been widely recognized that HOV lanes can carry more people than General Purpose (GP) lanes during peak hours. Kim conducted a study using micro-simulation models and found HOV lanes improved travel time significantly *(2)*. On the other hand, the research on the usage of HOV lanes conducted by Dahlgren indicates under some circumstances HOV lanes are less effective in reducing traffic delay *(3)*. In Kwon and Varaiya's studies *(4)*, HOV lanes were found to increase congestion in the Bay area HOV networks by underutilizing the lane capacities by about 400 vph. Many HOV facilities are underutilized when other GP lanes are congested. This may be partially because about 43% of carpoolers are members of the same household and exploitation of HOV lanes is restricted to some extent *(5)*.

To address these problems, High Occupancy Toll (HOT) lane concept was proposed as a potential means to mitigate traffic congestion and improve traffic mobility. Single Occupancy Vehicles (SOVs) are allowed to access to HOV lanes by paying a toll when the excess capacities of HOV lanes are available. Currently, there are about 1285.3 miles of HOV lanes in the US *(6)*. Successful operation of HOT lanes by fully exploiting their excess capacities will generate huge potential time savings and significantly mitigate traffic congestion. Therefore, HOT lanes have been increasingly recognized in research and practice as a viable measure to improve traffic operation efficiency.

The first HOT lane project was implemented on State Route 91 in Orange County, California in 1995. After that, three other states: Texas, Minnesota, and Colorado have implemented HOT lanes *(7)*. Although many studies have been conducted to evaluate the system performance of these projects *(8, 9,* and *10)*, few of them were focused on the development of optimized tolling strategies. An applicable and optimal tolling algorithm is essential to enhance the overall HOT lane system performance by adjusting traffic assignment rapidly and consistently. Therefore, we have been motivated to develop a new feedback-based tolling algorithm for dynamically optimizing HOT lane manipulation.

The next section briefly describes the state of the art regarding tolling strategies for practical HOT lane systems. It is followed by an in-depth analysis of major problems with these existing tolling approaches. Then, the details of the proposed tolling algorithm are presented in the methodology section. The VISSIM-based simulation tests and discussion on the performance of this tolling algorithm are described in the section following the methodology. The final section concludes this research effort and proposes further research topics.

2. STATE OF THE ART

Existing research on tolling algorithms for HOT lane operations is still in its early stage. In practice, rough dynamic tolling strategies have been executed for HOT lane operations. For example, for I-15 HOT lanes in San Diego, the basic price varies from $0.50 to $4.00 according to the time of day. The tolls may be adjusted in response to real time traffic conditions. The maximum value of $8.00 is employed for heavily congested situations *(11)*. For the I-394 MnPass Express lane in Minnesota, a similar pricing mechanism is implemented. The tolls are adjusted upward or downward to ensure the HOT lane flow rates at about 50-55 mph. The traffic density is applied as the detection input and the update interval for tolling is specified as 3 minutes. The toll ranges from 25 cents to $8 and averages $1 to $4 during rush hours *(12)*. Although these tolling approaches approximately realize traffic response-based toll adjustment, due to insufficient theoretical basis, it is hard to quantitatively achieve the optimal system performance.

Review of previous literature did not find a systematic and applicable approach for dynamically determining toll for HOT lane operations. Chu, Nesamani, and Benouar proposed a priority-based operation framework for HOV lane usages based on vehicle occupancy, type and toll rate *(6)*. But no further investigations were conducted on dynamic tolling strategies. Yin and Lou proposed two approaches for dynamic toll determination *(11)*. The first one is to employ the ramp metering control algorithm, ALINEA *(13)*, into the HOT lane management by changing metering rates to toll rates. The control logic is expressed as follows:

$$r(t+1) = r(t) + K \cdot (o(t) - o^*) \tag{1}$$

where, $r(t)$, and $r(t+1)$ are the toll rates at interval t and $t+1$, respectively; $o(t)$ is the measured occupancy; K is the regulator parameter; o^* is the desired occupancy of the HOT lane. The other approach is to utilize the route choice model, the Logit model, for tolling determination. Their major research work was concentrated on parameter estimation and model calibration using real-time traffic counts measured from both HOT and GP lanes. Although there are certain similarities among ramp metering control, route choice models and HOT tolling strategies, the unique characteristics of HOT lane operations can not be sufficiently accommodated by simply transplanting another control method. Hence, a more efficient tolling algorithm aiming at overall HOT lane operation optimization is needed.

3. PROBLEM STATEMENT

Correctly identifying and analyzing the primary malpractice of the existing tolling strategies is crucial to developing an optimum tolling algorithm to further enhance HOT lane system performance. Currently, although rough feedback control mechanisms are conceptually used for HOT lane operations based on either practical experience or existing ramp metering algorithms, it is difficult to quantitatively accomplish optimal HOT lane exploitation due to theoretical deficiency in tolling schemes. When the toll-rate change is improperly large or the toll update interval is too short, traffic allocation between HOT lanes and GP lanes may unstably overshoot and further cause serious flow fluctuation for both lanes. Conversely, inappropriately small toll-rate changes or long updated intervals may generate serious reaction delays and deteriorate system performance. To visualize the weakness of these tolling algorithms, an illustrative example is shown in Figure 1 quantifying their performance. The scenario is configured so that the current traffic speed on the HOT lane would be adjusted from 35 mph to 45 mph. Different response processes are performed by the different tolling algorithms. The underdamped tolling strategy may generate unfavorable flow fluctuation because of over-sensitive reaction although it can respond to traffic condition changes fast. The under-sensitive tolling algorithm may control traffic stably but cause severe response delays because of overdamped characteristics. In such tolling systems, hysteresis properties of traffic systems can not be sufficiently handled and will severely degrade overall system performance.

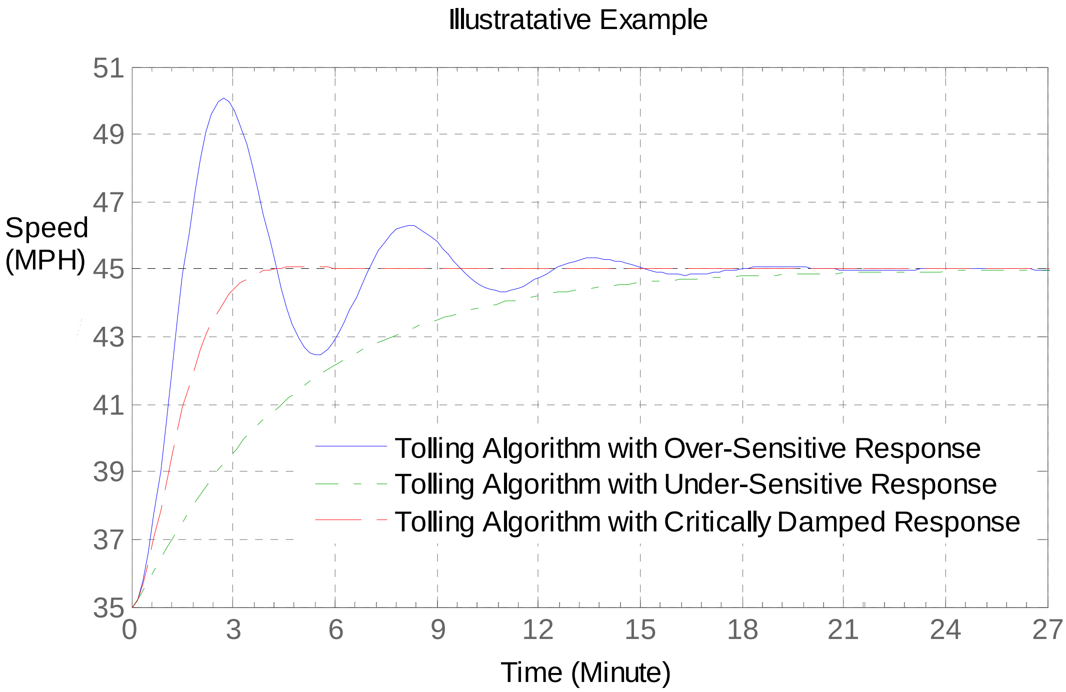

FIGURE 1 Comparisons of speed response on HOT lanes for different tolling algorithms.

Therefore, a new tolling algorithm is proposed to dynamically optimize the HOT lane operation using feedback control theory in this study. The operation objective is to enable the tolling algorithm to adaptively accommodate traffic variations fast and stably, as illustrated by the critically damped curve in Figure 1. Via dynamic toll adjustment, traffic assigned into HOT lanes is regulated to fully exploit the extra capacity without degrading operation conditions for HOVs. The details of the proposed methodology are described in the next section.

4. METHODOLOGY

4.1 Algorithmic Scheme

Practical experience on HOT lane operations and in-depth investiagtions of existing tolling approaches provide valuable insights into intrinsic problems in tolling optimization. From a macroscopic perspective, the toll can function as an adjustment lever to direct the traffic inflow to the HOT lane. As shown in Figure 2, based on traffic conditions on HOT and GP lanes, the tolling algorithm is executed and the toll is determined for the next interval. Then motorists make decisions whether to use the HOT lane according to the toll and traffic situations in the network. Finally, the desired number of vehicles access to the HOT lane, and the optimum system operation is achieved. However, due to the complicated nonlinear associations between the toll rate and the traffic flow entering the HOT lane, simple tolling algorithms may not satisfy the flexibility to control the traffic assignment. On the other hand, complex tolling algorithms are capable of adjusting traffic allocation competently, but are not easily implemented to meet practical needs. To address these problems, we propose an applicable tolling algorithm in this study. A second-order control scheme is explioted in this algorithm. First, using feedback control logic the ideal traffic flow ratio for optimal HOT lane utilization is calculated. Then, the optimal toll rate is estimated backward using the Logit model, one of the widely-recognized route decision models. By decomposing the calculation complexity, this new algorithm can satisfy the practicality and effectiveness required by the HOT lane system in reality.

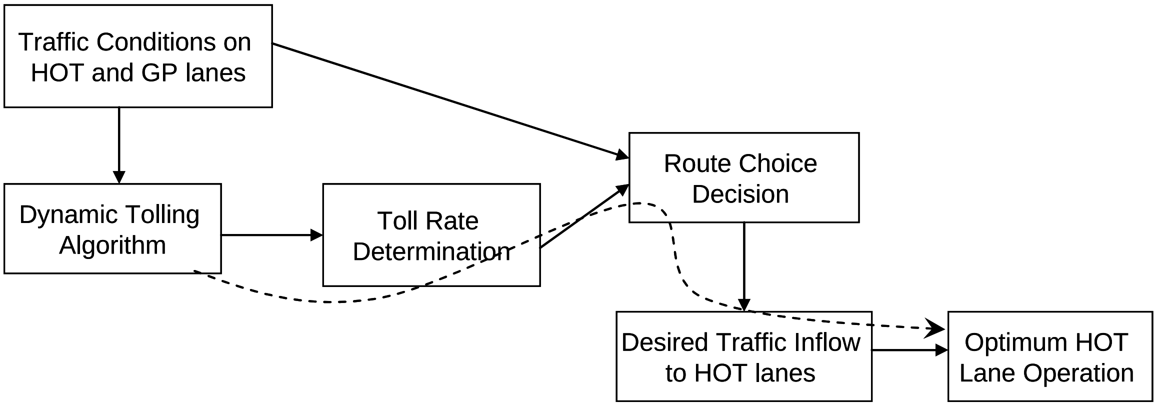

FIGURE 2 A schematic flow chart of the typical HOT lane operation.

4.2 Modeling HOT lane Utilization

In reality, motorists will make decisions on whether to pay for accessing the HOT lane based on the toll rate, and traffic conditions on the HOT and GP lanes. Such a decision-making process can be formulated by the commonly used Logit model.

To quantify the attractiveness of different lanes, the total costs, TC_i, for choosing lane type i is computed as

$$TC_i = \alpha * TT_i + TR_i \tag{2}$$

where, TT_i is the average travel time and TR_i is the toll rate for lane type i; α is the coefficient to convert TT_i into cash value. For the type of GP lane, the toll rate $TR_{GP}=0$. The travel distance is excluded in this equation due to its static attributes. Then, utility function U for each lane choice is calculated as

$$\begin{aligned} U_{HOT} &= \frac{1}{TC_{HOT}} = \frac{1}{\alpha * TT_{HOT} + TR_{HOT}} \\ U_{GP} &= \frac{1}{TC_{GP}} = \frac{1}{\alpha * TT_{GP}} \end{aligned} \tag{3}$$

where, U_{HOT} is the utility function of the HOT lane and U_{GP} is the utility function of the GP lane. Then the traffic assignment is modeled by the Logit model. The traffic flow, F_{HOT} , entering the HOT lane can be obtained

$$F_{HOT} = F_{total} * P_{HOT} = F_{total} * \frac{\exp(U_{HOT})}{\exp(U_{HOT}) + \exp(U_{GP})} = F_{total} * f(TR_{HOT}, TT_{HOT}, TT_{GP}) \qquad (4)$$

where, F_{total} is the total approaching traffic flow; P_{HOT} is the probability of choosing the HOT lane for each individual vehicles; also, from a macroscopic perspective, P_{HOT} denotes the flow ratio for HOT lane utilization. $f()$ is an abstract function that associates the independent variables, TR_{HOT}, TT_{HOT}, and TT_{GP} with dependent variable P_{HOT}. Due to the one-to-one transformation between TR_{HOT} and P_{HOT}, the toll rate can be calculated inversely,

$$TR_{HOT} = f^{-1}(F_{HOT} / F_{total}, TT_{HOT}, TT_{GP}) = f^{-1}(P_{HOT}, TT_{HOT}, TT_{GP}) \qquad (5)$$

where, $f^{-1}()$ indicates the corresponding inverse function. In this equation, the variables, TT_{HOT}, TT_{GP}, and F_{total} are measurable based on the traffic detection system. Therefore, if F_{HOT} is determined, the optimal toll rate TR_{HOT} can be backward computed. In reality, the approaching traffic flow F_{total} can be approximately regarded to be consistent between two consecutive update intervals. So the calculation of F_{HOT} can be simplified by computing P_{HOT}. In the following sections, a feedback-based piecewise control algorithm is exploited to calculate P_{HOT}, and then, after the related coefficients are calibrated, the toll rate can be estimated.

4.3 Feedback Control Mechanism

Feedback control is one simple yet effective control approach that has been widely applied in engineering and mathematic fields, such as an automobile speed control, satellites, robots, and industrial processes *(14)*. Enlightened by feedback control mechanisms applied in other fields, a feedback-based piecewise linear function is developed and utilized to calculate P_{HOT}, the optimum flow ratio for HOT lane utilization.

In many HOT lane projects the HOT lane speed is employed as one of critical Measures of Effectiveness (MOEs) to indicate the system operation status. For instance, in the Washington Sate Route 167 HOT lane pilot project, the HOT lane speed is required to be higher than 45 mph *(15)*. Therefore, in this study, the speeds of HOT and GP lanes are employed as feedback variables and the operation criteria is to maintain the HOT lane speed higher than 45 mph ensuring the HOVs' travel reliability. The control principle of the algorithm is to divide the HOT lane operation status into three manipulation zones based on the robustness of HOT lane speed, and then to develop the specific control strategy for each zone to achieve overall optimal system performance. Let V_{HOT} and V_{GP} denote the average speeds of HOT and GP lanes, respectively. Three manipulation zones are partitioned according to their robustness: the first zone is $V_{HOT} >$ 50 mph, which indicates sufficient HOT lane capacities are available, and the toll needs to be rapidly adjusted in a large range; the second zone is $50 \geq V_{HOT} > 45$ mph, which shows the traffic on the HOT lane is close to a critical state, and the toll should be maintained at the same level; the third zone is $V_{HOT} \leq 45$ mph, the overflowing traffic degrades the HOT lane performance, and the toll must be increased fast.

Different feedback control mechanisms are adopted for these three zones and can be formulated as:

$$P_{HOT}(t+1) = P_{HOT}(t) + \Delta P_{HOT}(t) = P_{HOT}(t) + \begin{cases} b_1 + k_1(V_{HOT}(t) - V_{GP}(t)) & V_{HOT}(t) > 50 \\ sign*[b_2 + k_2(V_{HOT}(t) - V_{GP}(t))] & 50 \geq V_{HOT}(t) > 45 \\ k_3(V_{HOT}(t) - 45) & V_{HOT}(t) \leq 45 \end{cases} \quad (6)$$

where, $P_{HOT}(t+1)$ and $P_{HOT}(t)$ are the flow ratios for HOT lane usage at time interval t and $t+1$; $\Delta P_{HOT}(t)$ is the feedback increment; b_1, b_2, k_1, k_2, and k_3 are the parameters indicating control intensities of feedback quantities; $V_{HOT}(t)$ and $V_{GP}(t)$ are the average traffic speeds on HOT and GP lanes at time interval t, respectively; *sign* is a variable describing the changing pattern of P_{HOT}, and is defined as:

$$sign = \begin{cases} 1 & P_{HOT}(t-1) > P_{HOT}(t) \\ 0 & P_{HOT}(t-1) = P_{HOT}(t) \\ -1 & P_{HOT}(t-1) > P_{HOT}(t) \end{cases} \quad (7)$$

In Equation (6), when $V_{HOT} > 50$ mph, the speed difference between the HOT and GP lanes is employed as the feedback variable. The feedback increment, $\Delta P_{HOT}(t)$, is represented by a linear function, $b_1 + k_1(V_{HOT}(t) - V_{GP}(t))$. Such a feedback layout can effectively reflect traffic conditions on HOT and GP lanes and provide sufficient flexibilities to ameliorate feedback mechanism. When $50 \geq V_{HOT} > 45$ mph, besides an analogous feedback function, the indication variable, *sign,* is used to reflect the changing tendency of $P_{HOT}(t)$. Consequently, the feedback increment, $\Delta P_{HOT}(t)$, presents an alternative scheme to preserve HOT lane operation stability. When $V_{HOT} \leq 45$ mph, $P_{HOT}(t)$ decreases directly by adding a negative item, $k_3(V_{HOT}(t) - 45)$.

According to this feedback-based piecewise control algorithm, the optimum traffic flow ratio for HOT lane utilization can be calculated iteratively for each time interval. Then the appropriate toll rate can be backward estimated. The details of parameter calibration and toll estimation are presented in the next section.

4.4 Toll Rate Estimation

The traffic speeds can be measured in real-time for HOT and GP lanes by the detection system. Then the travel times are calculable for both lanes. Based on Equation (6) and (5) the toll rate can be estimated after the related parameters are determined

In Equation (6), five parameters, b_1, b_2, k_1, k_2, and k_3, need to be determined. These parameters denote the acting intensity of feedback quantities and have to be calibrated separately according to different control strategies. For instance, when the HOT lane speed is larger than 50 mph, the redundant capacity on the HOT lane is available. To optimize the overall traffic operation, traffic allocation needs to be adjusted rapidly. Based on the traffic speed variation range and the efforts of trial and error, the parameters b_1, and k_1 are set as b_1=0.075, and k_1 = 0.005. Similarly, the other parameters can be calculated as b_2=0.024, k_2=0.0012 and k_3 =0.03. The reasonableness of these values can be demonstrated by the following calculation case:

$$\begin{cases} V_{HOT} = 53; & V_{GP} = 48; \quad then \quad \Delta P_{HOT} = 0.075 + 0.005*(53-48) = 10\% \\ V_{HOT} = 53; & V_{GP} = 28; \quad then \quad \Delta P_{HOT} = 0.075 + 0.005*(53-28) = 20\% \\ V_{HOT} = 48; & V_{GP} = 35; \quad then \quad \Delta P_{HOT} = 0.024 + 0.0012*(48-35) = 3.9\% \\ V_{HOT} = 40; & \qquad\qquad\quad then \quad \Delta P_{HOT} = 0.03*(40-45) = -15\% \end{cases} \tag{8}$$

Assuming $V_{HOT} = 53$ mph, the traffic on the HOT lane operates in a robust status, so the feedback increment, ΔP_{HOT}, is updated at a higher changing rate, such as 10% to 20% depending on the GP lane speed; when $V_{HOT} = 48$ mph, traffic speed of the HOT lane is close to the critical speed, 45 mph, and thus should be maintained at a consistent level, the flow ratio for HOT lane utilization changes slightly. For example, when $V_{GP} = 35$ mph, ΔP_{HOT} is only 3.9% (increase or decrease is associated with the variable, *sign*, in Equation (7)); when $V_{HOT} = 40$ mph, the HOT lane speed is lower than the critical speed, as a result, the flow ratio for HOT lane utilization needs to decrease sharply without considering the GP lane speed. Note that these values are not unique solutions for these parameters. Different sets of values may achieve analogous control results. Following the control principle proposed in this study, parameter calibration can be strengthened to meet the specific requirements of other applications.

After P_{HOT} is calculated for the next interval using Equation (6), the toll rate TR_{HOT} can be estimated by the inverse function in Equation (5). Further calculation is conducted to embody this process. The toll rate TR_{HOT} can be obtained:

$$TR_{HOT} = f^{-1}(P_{HOT}, TT_{HOT}, TT_{GP}) = \frac{1}{\dfrac{1}{\alpha \cdot TT_{GP}} - \ln(\dfrac{1-P_{HOT}}{P_{HOT}})} - \alpha \cdot TT_{HOT} \tag{9}$$

where, the coefficient α needs to be determined. In our study, the capital-to-travel time ratio of \$11.7 per hour is applied to compute α, e.g. α = 11.7 dollar / hour = 0.325 cent / second. This ratio value was obtained from the traffic survey in the greater Seattle area. Actually, this coefficient indicates motorists' willingness to pay for using HOT lanes and is closely associated with many particular factors, such as local economic conditions, traffic patterns, geographic characteristics, population distribution and so on. Some studies were conducted to quantify these factors' impacts on HOT lane usage *(16, 17,* and *18)*. Findings show these factors are location-specified variables, and no uniform settings are applicable for manifold practical applications. More detailed discussions are beyond this paper's scope. When this tolling algorithm is used in other applications, the coefficient α should be recalibrated to adapt to different situations.

Based on Equation (6) and (9), it is straightforward to estimate the toll rate for the next interval. To verify the effectiveness of the proposed tolling algorithm, VISSIM-based simulation experiments were conducted in the next section.

5. SIMULATION EXPERIMENTS AND DISCUSSION

5.1 Simulation Model Development

Although VISSIM 4.30 is widely utilized for freeway modeling due to its competent capabilities of simulating various common transportation operations *(19)*, it can not sufficiently handle the

HOT lane operation issues using its built-in modules. In this study, an independent functional module is developed to realize HOT lane simulation through the VISSIM Component Object Model (COM) application interfaces. Microsoft Visual Basic is used as the external program to develop the HOT lane module. And the proposed tolling algorithm is implemented in this module. A simulation model is established to emulate the HOT lane operation for the Washington State Route 167 HOT lane pilot project.

Five HOT lane sections on northbound SR-167 from SW 15th Street in Auburn to Interstate 405 in Renton, WA, are configured in the simulation model by following the exact geographic features in reality, including, the location of on-ramps and off-ramps, freeway curvature and length, weaving sections, the number of lanes and so on *(15)*. A sketch map for the overall simulation network is shown in Figure 3. Selection of these five HOT lane sections is determined by the fact that they represent diversified HOT lane features. For example, 2 off-ramps and 3 on-ramps are distributed along HOT Lane Segment 1, and its length is about 2.8 miles, and HOT Lane Segment 2 is about 1.3 miles with 1 off-ramp and 1 on-ramp, etc. The morning-peak hours of 6:00 to 9:00am is chosen as the simulation time period. Three kinds of vehicles are adopted to represent a general traffic composition: SOVs, HOVs and trucks. Dynamic traffic assignment is utilized to strengthen the practicality and validity of the simulation model. Based on the observed ground-truth data, including traffic volume and speed, the simulation model is well calibrated to properly reproduce the existing traffic conditions. More details of this HOT lane module development and simulation model calibration are described in *(20)*.

To acquire accurate speed data needed by the tolling algorithm, each HOT lane section has virtual loop detectors deployed evenly along the HOT and GP lanes in the simulation model. Based on these real-time speed data, the space mean speed is calculated for the travel time estimation and toll determination. Based on network characteristics of this HOT lane system and the efforts of trial and error, the toll update interval is specified as 5 minutes to balance the control stabilization and timeliness. Additionally, to minimize the randomness of simulation results and enhance simulation models' credibility, a total of 7 simulation runs were conducted, each with a different random seed arbitrarily selected. The integrated results from these simulation runs are considered statistically reliable and unbiased.

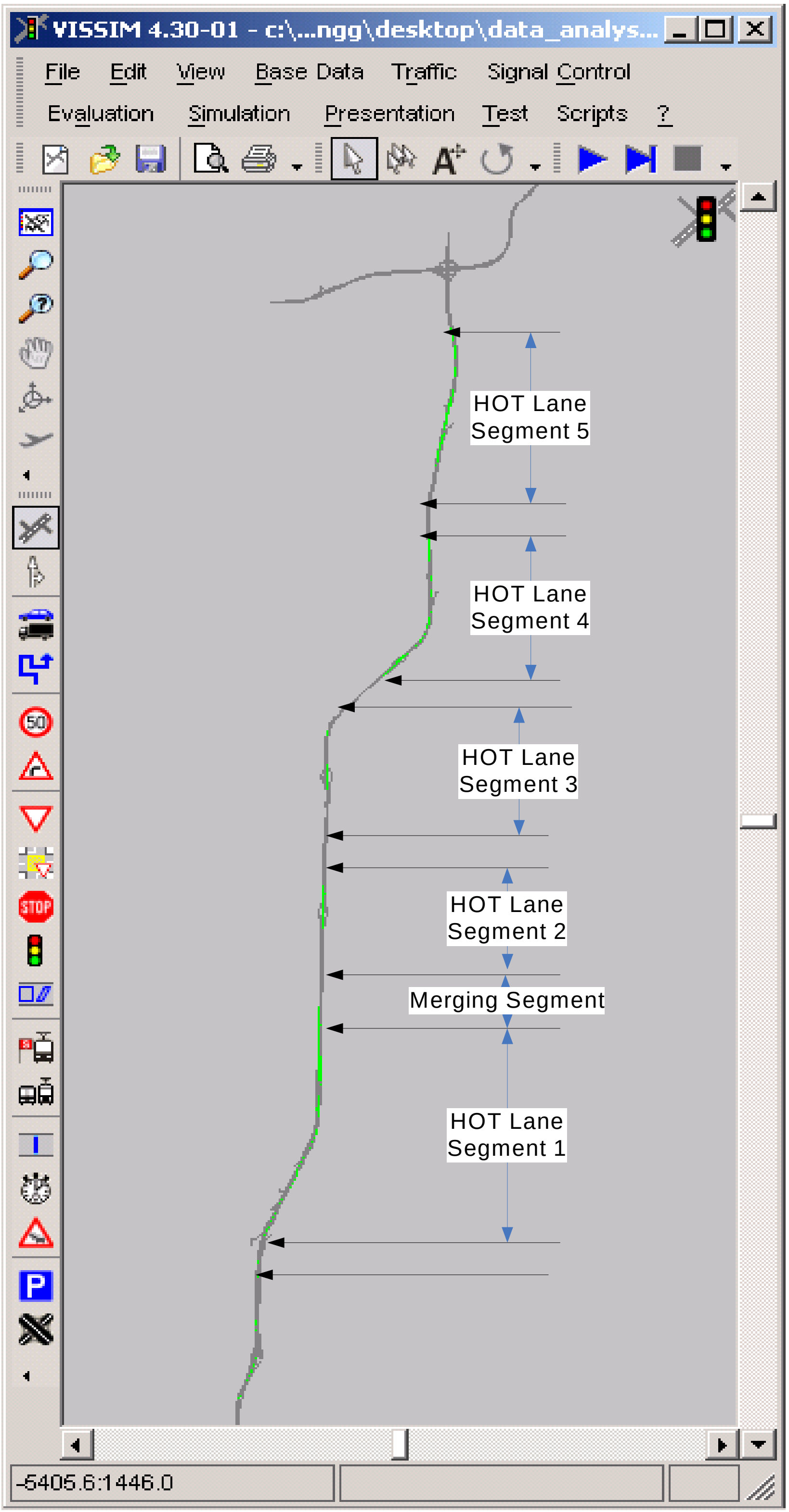

FIGURE 3 A sketching overview of simulation network.

5.2 Simulation Test and Discussion

To fully investigate the robustness and applicability of the proposed tolling algorithm, simulation tests were conducted under various traffic demands. Based on the calibrated simulation model, the current peak-hour traffic demands from 6:00-9:00am serve as the master data, and then traffic volumes change in 10% increments from 80% to 140% of master data. Such wide-ranging test conditions provide a reliable platform to demonstrate the effectiveness of the tolling algorithm and quantify HOT lane system performance.

First, simulation experiments were conducted under existing traffic conditions. The integrated simulation results are summarized in Table 1, including travel time, throughputs, and space-mean speeds for the entire HOT lane system. Also, to facilitate the comparison of traffic throughputs achieved under HOV lane operations against these under HOT lane operations, an improvement ratio is defined. The throughput difference between the HOT lane system and HOV lane system is divided by the throughputs of the HOV lane system. Similarly, an analogous variable is defined for traffic speed. Table 1 shows these variables as the improvement for throughputs and speeds. As we can see, for example, Lane Segment 2, under HOV lane operations, the average speed of the GP lanes is 41.2 MPH, and that of HOV lane is 60.0 MPH. The average speed for both GP and HOV lanes is about 43.6 MPH. Under HOT lane operations, the number of vehicles choosing the HOT lane increase from 1574 to 2430 within 3 hours, and, consequently, the average speed of the GP lanes increase to 53.5 MPH. However, the HOT lane speed stands at the same level, about 59.2 MPH and the overall average speed of both HOT and GP lanes increases to 55.0 MPH from 43.6 MPH, an improvement of 26.2%. The total throughputs maintain the same level although there is a slight increase from 9153 to 9226. This is because although there is congestion under HOV lane operation, the existing traffic system is capable of handling all through vehicles within present traffic demands. Hence, there is not significant enhancement in terms of traffic throughputs. Also analogous analysis can be conducted for on-ramp and off-ramp traffic for Lane Segment 2.

For other HOT lane segments, similar results can be obtained from Table 1. The overall traffic efficiency is improved remarkably under HOT lane operations. Note that for HOT Lane Segment 5, the traffic operates reasonably well under the present HOV lane system. The average speed for both GP and HOV lanes is about 53.9 MPH. Therefore, there are not considerable improvements achieved under HOT lane operations.

Further simulation tests were conducted on broadly changing traffic demands. Based on the present traffic condition, six additional test scenarios are proposed with the demands distributing from 80% to 140% of the current demand. The simulation results for HOT Lane Segment 2 are shown in Table 2 to represent a typical HOT lane operation. As can been seen from Table 2, under HOV lane operations the maximum throughputs that can be efficiently handled by the existing infrastructure is about 9153 vehicles with the current demand. When the demand increases, overall traffic conditions deteriorate, and traffic speeds and throughputs decrease notably. However, under HOT lane operations, due to the enhanced traffic capacities, the maximum throughputs increase to 9489 vehicles corresponding to 110% of the current demand. The improvements of traffic throughputs and speeds accomplished by HOT lane operations are progressively noticeable with the demand increasing. The maximum improvements are 11.8% and 61.5% for traffic throughputs and speeds, respectively, corresponding to 130% of the current demand. Note that immunizing from the negative impacts of increased demands, the HOT lane speed is preserved in the desired range. The minimum speed is about 47.1 MPH, that is slightly higher than 45 MPH, the HOT lane speed criteria. These analytical results clearly indicate enhanced system performance of the HOT lane system and further clarify the proposed tolling algorithm is capable of optimizing the overall traffic operation and ensuring travel reliability on the HOT lane under various traffic conditions.

TABLE 1 Integrated Simulation Results under Existing Traffic Demands From 7 Simulation Runs

Simulation Time Period: 6:00-9:00 Am		HOV Operation			HOT Operation			Improvement	
		TT[a]	TP[b]	SP[c]	TT	TP	SP	TP	SP
Merging Area 1		57.9	7747	34.2	64.3	7732	30.8	-0.2%	-9.9%
HOT Segment 1: Length = 2.8 M	GP+HOV/HOT	230.0	5588	43.6	176.1	5586	56.9	0.0%	30.6%
	GP Lane	242.0	4677	41.4	180.8	2573	55.4	-45.0%	33.8%
	HOV/HOT Lane	168.4	912	59.5	172.1	3013	58.3	230.5%	-2.1%
	On-Ramps	71.7	5401	24.5	32.7	5445	52.2	0.8%	113.0%
	Off-Ramps	22.5	2712	34.2	20.7	2684	36.9	-1.0%	7.9%
Merging Area 2		44.6	10142	48.7	44.6	10210	48.7	0.7%	0.0%
HOT Segment 2: Length = 1.3 M	GP+HOV/HOT	107.3	9153	43.6	85.0	9226	55.0	0.8%	26.2%
	GP Lane	113.3	7579	41.2	87.3	6796	53.5	-10.3%	29.8%
	HOV/HOT Lane	77.9	1574	60.0	78.9	2430	59.2	54.4%	-1.2%
	On-Ramps	42.8	1588	43.6	38.7	1592	48.0	0.2%	10.0%
	Off-Ramps	43.6	893	48.6	39.0	899	53.1	0.7%	9.3%
Merging Area 3		40.5	10695	44.0	44.0	10789	40.5	0.9%	-8.0%
HOT Segment 3: Length = 1.2 M	GP+HOV/HOT	86.0	8511	49.9	74.7	8614	57.4	1.2%	15.1%
	GP Lane	88.5	7272	48.5	75.5	7151	56.9	-1.7%	17.2%
	HOV/HOT Lane	71.3	1239	60.2	71.0	1463	60.0	18.1%	-0.3%
	On-Ramps	35.1	1573	52.5	30.8	1575	57.2	0.1%	9.0%
	Off-Ramps	39.8	2092	47.6	38.1	2109	49.3	0.8%	3.7%
Merging Area 4		37.5	10033	47.1	33.5	10155	52.7	1.2%	11.9%
HOT Segment 4: Length = 2.4 M	GP+HOV/HOT	225.4	7288	37.7	149.8	7435	56.8	2.0%	50.5%
	GP Lane	239.7	6207	35.5	152.1	4852	55.9	-21.8%	57.6%
	HOV/HOT Lane	143.5	1081	59.3	145.2	2583	58.6	138.9%	-1.2%
	On-Ramps	40.4	3310	44.7	33.3	3352	53.6	1.3%	19.9%
	Off-Ramps	34.2	2715	38.2	25.5	2760	49.9	1.7%	30.6%
Merging Area 5		35.2	9905	56.2	41.4	10062	47.8	1.6%	-15.0%
HOT Segment 5: Length = 1.4 M	GP+HOV/HOT	92.1	7445	53.9	86.4	7581	57.4	1.8%	6.6%
	GP Lane	94.7	5783	52.4	88.0	4583	56.4	-20.8%	7.6%
	HOV/HOT Lane	83.0	1663	59.8	83.9	2998	59.1	80.3%	-1.1%
	On-Ramps	16.0	2475	50.1	14.3	2476	55.8	0.0%	11.4%
	Off-Ramps	53.3	2366	44.5	48.6	2410	48.8	1.9%	9.6%

[a] Travel Time (Second), [b] Throughputs, [c] Speed (Mile Per Hour)

TABLE 2 Integrated Simulation Results for HOT lane Segment 2 under Various Traffic Demands From 7 Simulation Runs

Simulation Time Period: 6:00-9:00 Am		HOV Operation			HOT Operation			Improvement	
		TT[a]	TP[b]	SP[c]	TT	TP	SP	TP	SP
80%	GP+HOV/HOT	80.7	7976	57.9	80.5	7945	58.1	-0.4%	0.2%
	GP Lane	81.4	6384	57.4	81.2	6287	57.6	-1.5%	0.2%
	HOV/HOT Lane	77.8	1592	60.0	77.7	1659	60.1	4.2%	0.1%
	On-Ramps	35.7	1321	51.9	35.5	1321	52.1	0.1%	0.5%
	Off-Ramps	38.5	776	53.9	38.4	774	53.9	-0.3%	0.0%
90%	GP+HOV/HOT	83.2	8576	56.1	81.7	8560	57.2	-0.2%	1.9%
	GP Lane	84.4	6985	55.3	82.8	6568	56.4	-6.0%	1.9%
	HOV/HOT Lane	77.9	1591	60.0	78.0	1992	59.9	25.2%	-0.2%
	On-Ramps	37.6	1443	49.3	36.7	1444	50.5	0.0%	2.5%
	Off-Ramps	38.9	833	53.3	38.8	832	53.5	-0.1%	0.3%
100%	GP+HOV/HOT	107.3	9153	43.6	85.0	9226	55.0	0.8%	26.2%
	GP Lane	113.3	7579	41.2	87.3	6796	53.5	-10.3%	29.8%
	HOV/HOT Lane	77.9	1574	60.0	78.9	2430	59.2	54.4%	-1.2%
	On-Ramps	42.8	1588	43.6	38.7	1592	48.0	0.2%	10.0%
	Off-Ramps	43.6	893	48.7	39.0	899	53.2	0.7%	9.2%
110%	GP+HOV/HOT	134.6	9117	34.7	93.4	9489	50.0	4.1%	44.0%
	GP Lane	145.7	7621	32.1	98.7	6547	47.3	-14.1%	47.7%
	HOV/HOT Lane	78.1	1496	59.8	81.8	2942	57.1	96.7%	-4.5%
	On-Ramps	47.5	1691	39.1	41.8	1700	44.3	0.5%	13.3%
	Off-Ramps	53.4	889	39.1	39.3	913	52.8	2.6%	35.0%
120%	GP+HOV/HOT	183.1	8764	25.5	117.6	9279	39.7	5.9%	55.8%
	GP Lane	202.2	7410	23.1	127.8	6643	36.5	-10.4%	58.2%
	HOV/HOT Lane	78.4	1354	59.6	88.5	2636	52.8	94.7%	-11.4%
	On-Ramps	55.0	1819	33.8	46.8	1812	40.4	-0.4%	19.5%
	Off-Ramps	75.3	897	27.6	46.6	924	45.4	3.0%	64.9%
130%	GP+HOV/HOT	211.5	8330	22.1	131.0	9311	35.7	11.8%	61.5%
	GP Lane	234.8	7087	19.9	146.0	6350	32.0	-10.4%	60.8%
	HOV/HOT Lane	78.9	1243	59.2	96.3	2961	48.5	138.2%	-18.0%
	On-Ramps	75.4	1949	24.9	57.1	1947	34.4	-0.1%	38.5%
	Off-Ramps	90.3	893	23.0	52.3	942	41.2	5.6%	79.2%
140%	GP+HOV/HOT	255.2	7859	18.3	157.6	8277	29.6	5.3%	62.0%
	GP Lane	284.3	6747	16.4	176.8	6093	26.4	-9.7%	60.8%
	HOV/HOT Lane	79.1	1112	59.1	99.2	2184	47.1	96.3%	-20.3%
	On-Ramps	103.0	2073	18.2	59.2	1910	31.5	-7.8%	73.3%
	Off-Ramps	112.2	846	18.5	60.8	857	35.5	1.3%	92.1%

[a] Travel Time (Second), [b] Throughputs, [c] Speed (Mile Per Hour)

Visualized speed comparisons between the HOV/HOT and GP lanes under different operation scenarios confirm the analytical results achieved above. For instance, Figure 4 shows the comparison curves of traffic speed aggregated in 5-minute intervals for the HOV and GP lanes of Lane Segment 1 under HOV lane operations with the current traffic demand. And Figure 5 shows the corresponding speed curves under HOT lane operations. The changing patterns of the GP lane speed in both figures indicates that by employing the tolling algorithm proposed in this study, the excess capacities of the HOT lane are effectively utilized and the GP lane speed is considerably improved, and meanwhile, there are not obvious negative impacts on the HOVs because the desired travel speed and reliability are preserved. Because the HOT and GP lane speeds are relatively consistent, and the toll is stable around $0.10 to $0.20 for SOVs to use this HOT lane. Similar comparisons were conducted with the increasing demands. Figure 6 and 7 illustrate the speed comparison curves for the HOV/HOT lane and the GP lane with 140% of the current demand. We can see that under HOV lane operations although the HOV lane speed maintains at the ideal level, traffic congestion generated by the dramatic increase of traffic demands, severely corrupts the GP lane speed. This problem can be partially solved under HOT lane operations. Some SOVs are discharged through the HOT lane and the GP lane speed is improved to some extent. Figure 7 shows the HOT lane speed was suitably adjusted to adapt to the changing traffic conditions and optimize the overall traffic operations. Figure 8 demonstrates the changing pattern of the corresponding toll rate. Operated by the proposed tolling algorithm, the toll rate is self-adaptive to traffic conditions on GP and HOT lanes. For example, a maximum toll rate of about $3.90 per mile is immediately applied when the HOT lane speed drops to 30 MPH, then the HOT lane condition is improved rapidly, and the toll rate is back to the normal value, e.g. $0.5 per mile. Such matching patterns between the toll rate and HOT lane speed indicate the appropriate sensitivity and robustness of the proposed tolling algorithm.

In general, the simulation results demonstrate that the proposed tolling algorithm performs reasonably well under various demand scenarios. The overall traffic efficiency is enhanced significantly by optimizing traffic operations on HOT and GP lanes systematically. Meanwhile, travel speed and reliability of HOVs are favorably preserved to satisfy the operation requirements.

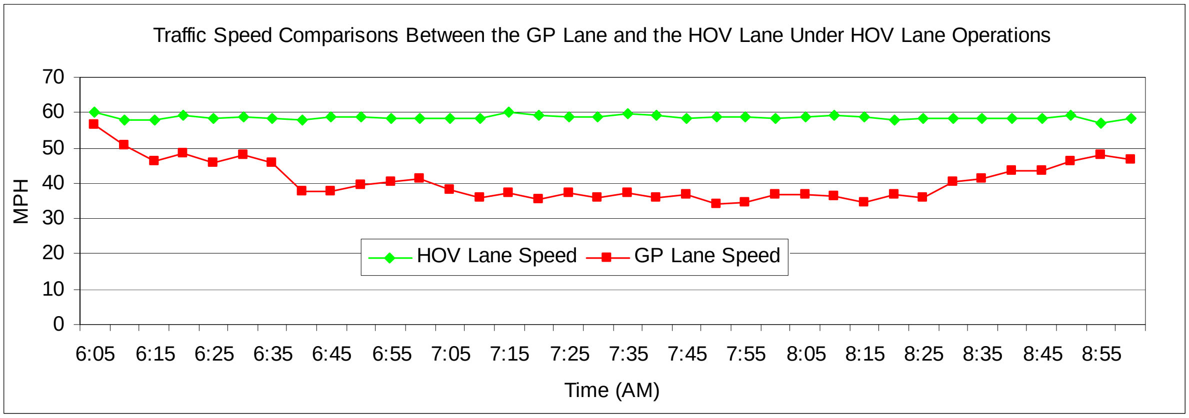

FIGURE 4 Traffic speed comparisons between the GP and HOV lanes for Lane Segment 1 under HOV lane operations with the current demand.

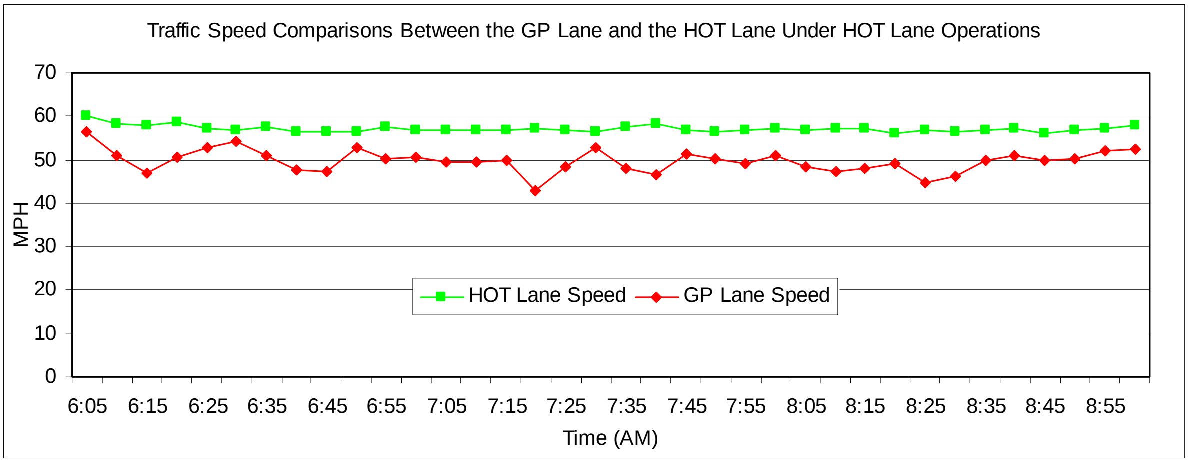

FIGURE 5 Traffic speed comparisons between the GP and HOT lanes for Lane Segment 1 under HOT lane operations with the current demand.

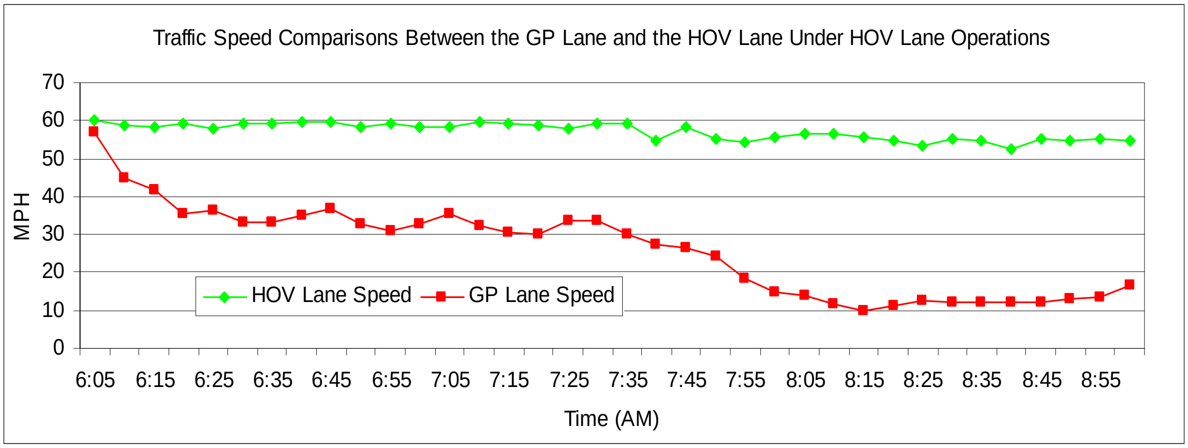

FIGURE 6 Traffic speed comparisons between the GP and HOV lanes for Lane Segment 1 under HOV lane operations with 140% of the current demand.

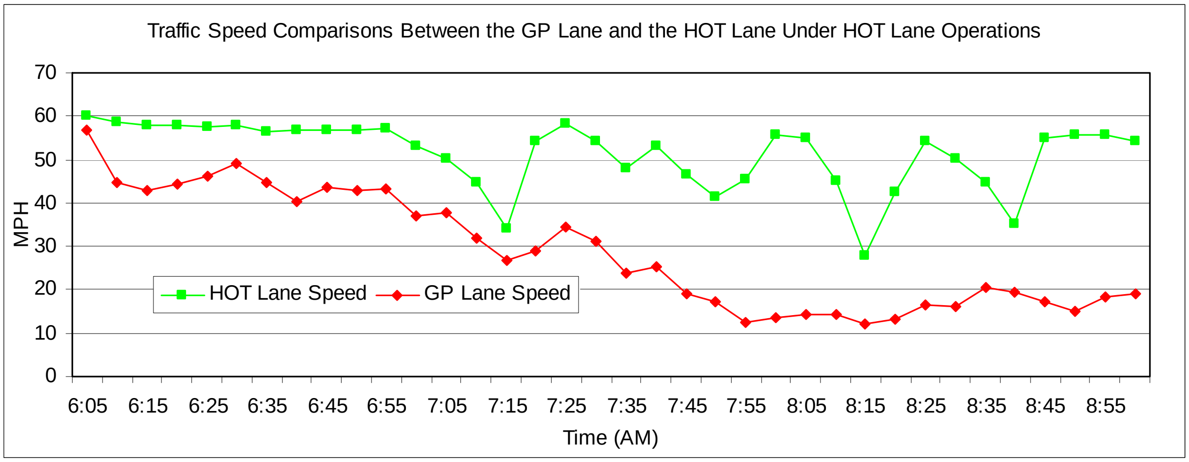

FIGURE 7 Traffic speed comparisons between the GP and HOT lanes for Lane Segment 1 under HOT lane operations with 140% of the current demand.

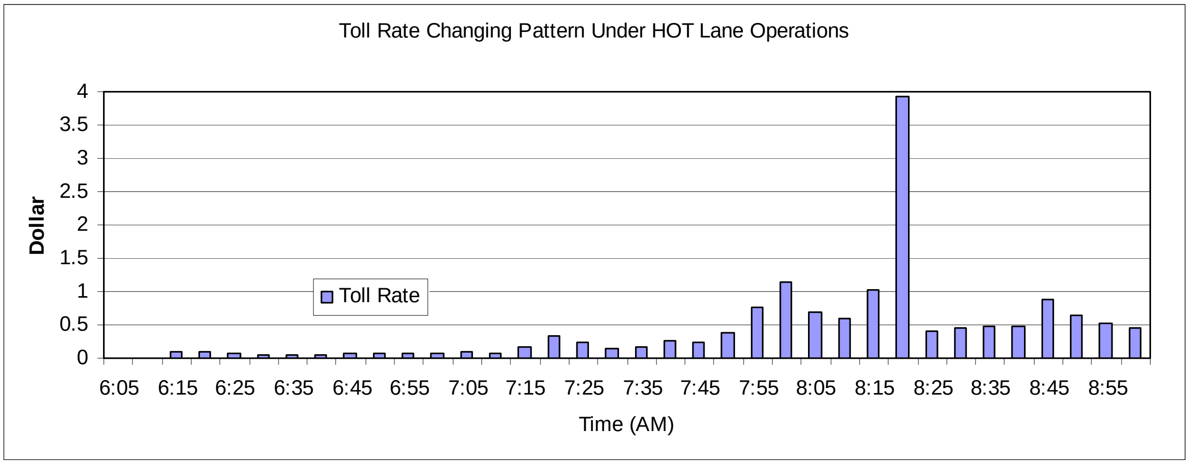

FIGURE 8 Toll rate changing pattern for Lane Segment 1 under HOT lane operations with 140% of the current demand.

6. CONCLUSIONS

The HOT lane concept has been increasingly recognized and accepted as a viable measure to mitigate freeway congestion and improve travel time reliability. Optimized HOT lane management can yield significant economic returns and social benefits. Four states, California, Texas, Minnesota, and Colorado, have implemented HOT lane projects. However, it is difficult to quantitatively accomplish optimal HOT lane exploitation due to theoretical deficiency in these tolling schemes, although rough traffic-response-based tolling algorithms are applied to HOT lane operations based on practical experience. Therefore, developing an applicable and optimum tolling algorithm to enhance HOT lane system performance is of practical significance.

Two major problems with the inferior tolling strategies degrade HOT lane system performance. First, the under-sensitive tolling algorithm is incapable of handling the hysteresis properties of traffic systems and may cause severe response delays. Secondly, unfavorable flow fluctuation on HOT and GP lanes may be generated agitating traffic operations due to the over-sensitive characteristics of the imperfect tolling strategies. To address these problems, a new feedback-based tolling algorithm is developed to dynamically optimize HOT lane operations. A second-order control scheme is exploited in this algorithm. Based on the traffic speed conditions and toll changing patterns, the optimum flow ratio for HOT lane utilization is calculated using feedback control theory. Then the proper toll rate is backward estimated using the Logit model. By decomposing the calculation complexity, this tolling algorithm can satisfy the practicality and effectiveness required by the HOT lane system in reality.

VISSIM-based simulation tests were conducted to examine the proposed tolling algorithm performance. An external HOT lane module was developed using Microsoft Visual Basic to implement the algorithm. And five HOT lane sections on southbound SR 167 corridor in Washington State were simulated. The simulation results show this feedback-based tolling algorithm performs reasonably well under various traffic demands. Through optimal toll adjustment, SOVs are regulated to fully exploit the excess capacity of the HOT lane without degrading operation conditions for HOVs. Overall traffic efficiencies are improved significantly.

Although the proposed tolling algorithm demonstrated the favorable performance for operating HOT lane systems, further improvements are desired for extensive applications through fusing lane occupancy and flow rate into the feedback quantity to strengthen the robustness of the algorithm.

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